Hypothesis Testing: Z-Test

Concepts

1. In general, statistics does not allow you to prove anything is true, but instead allows you to show that things are probably false. So when we do hypothesis testing, the **null hypothesis** H_0 is something that we want to show is false and the **alternative hypothesis** H_1 is something that you want to show is true. For example, to show that a drug cures cancer, the null hypothesis would be that the drug does nothing and the alternative hypothesis would be that the drug does help cure cancer.

   A **type 1 error** is rejecting a true null which means that in our example, saying a drug cures cancer when it doesn't. A **type 2 error** is failing to reject a false null which means in our case as saying that the drug doesn't do anything when it does. The **significance level** α is the probability of making a type 1 error. The **power** $1 - \beta$ is 1 minus the probability of making a type 2 error.

Examples

2. When Thanos snapped his fingers, everyone had a $p = 0.5$ chance of disintegrating. I think that this probability was much lower for the original Avengers. Out of the 6 of them, no one got disintegrated. Can you reject the null hypothesis that there was a $p = 0.5$ chance of each of them disintegrating with an $\alpha = 0.05$?

3. An infomercial claims that a miracle drug will cause you to grow all your hair back. There are 25 brave participants and surprisingly 7 people regrew their hair. If normally 10% of people regrow their hair, can you say that this drug worked?

Problems

4. (True story) A woman claimed that she could tell whether milk or tea was added first to a cup. She was given 4 cups with milk added first then tea and 4 cups of the opposite. She guessed all 8 correctly. Let the null hypothesis be that she guesses randomly and alternate hypothesis be that she actually can tell with $p > 0.5$. Can we say that she has this ability with $\alpha = 0.05$?

5. You flip a coin 100 times and get 55 heads. Can you say that it is biased towards heads? (use $\alpha = 0.05$)

6. An infomercial claims that a miracle drug will cause you to grow all your hair back. There are 100 brave participants and this time 20 people regrew their hair. If normally 10% of people regrow their hair, can you say that this drug worked?